

Bulk viscosity of a gas of neutrinos and coupled scalar particles, in the era of recombination

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Bulk viscosity may serve to damp sound waves in a system of neutrinos coupled to very light scalar particles, in the era after normal neutrino decoupling but before recombination. We calculate the bulk viscosity parameter in a minimal scheme involving the coupling of the two systems. We add some remarks on the bulk viscosity of a system of fully ionized hydrogen plus photons.

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INTRODUCTION

It has long been known that a simple form of bulk viscosity is a potential source of dissipation in mixtures of relativistic and nonrelativistic fluids. For the cosmologically interesting case of photons and matter in the era just before recombination, where shear viscosity does indeed strongly damp waves for wave-lengths up to some critical length, Weinberg [1] has given the expression for the bulk viscosity parameter,

$$\zeta = 4aT^4\tau_\gamma \left[\frac{1}{3} - \left(\frac{\partial P}{\partial \rho} \right)_n \right]^2. \quad (1)$$

Here τ_γ is a mean free time for photon in the medium and the electron (and proton) number density n is held fixed in the derivative. Since the derivative in (1) is of the order $(1/3 - 10^{-10})$ in the early universe, in view of the large photon/electron ratio, the effect is absolutely negligible in this system.

For what comes later in this paper, we comment on the parameter, τ_γ . The development that led to (1) was based in part on the solution by Thomas [2] of a relativistic transport equation for photons. However, Thomas' treatment included only photon absorption and emission. When Compton scattering is the interaction mechanism, it appears that τ_γ should be the mean free time for Compton scattering weighted by an efficiency factor for energy transfer, that is to say, multiplied by a number of order of the number of collisions required for temperature equilibration of a system which initially has different temperatures for the matter and for the radiation. When the scattering mechanism is Compton scattering and the system is near the recombination temperature this is a large number indeed.

The motivation for the present paper is the possible existence of a scalar field, ϕ , with mass smaller than neutrino masses and coupled to neutrinos. Over time, theorists have found a number of reasons to entertain such a possibility [3]- [8]. The most stringent bounds on the coupling strength, at least until recently, have come from supernovae [9]- [13]. Recently Beacom, Bell and Dodle-son [14] demonstrated the rather surprising result that, staying within these bounds, processes $\nu + \nu \rightarrow \phi + \phi$

can remove most of the neutrinos from the universe as the temperature declines into the region in which the neutrinos become non-relativistic, a region that roughly corresponds to the recombination region itself. However Hannestad [15] pointed out that the acoustic oscillations of the fluid of interacting neutrinos and ϕ 's in this picture could alter the CMB angular power spectrum in ways that conflict with the data ¹.

Beginning from this observation, Raffelt and Hannestad [17] have given coupling constant limitations that a true free-streaming requirement would put on the $\nu - \phi$ coupling. But more detailed calculations of the effect of ν scattering, annihilation or decay on the CMB and the matter power spectrum appear to show that the landscape of coupling constant limitations is considerably more complicated than dictated by the "free-streaming" criterion, and, indeed, that in some coupling schemes, parameters that give a "tightly coupled" $\nu - \phi$ fluid at the recombination temperature are compatible with the data [18].

The first purpose of the present paper is to point out a possible role for bulk viscosity in controlling the size of neutrino density fluctuations under some conditions. Clearly the region of application would be in between the tightly coupled case and the free streaming case. The origin of this dissipation is qualitatively similar to that which underlies (1). We have two gases, one completely relativistic, the other in the relativistic-non-relativistic transitional region. Thus if we apply an adiabatic compression, they change temperature by different amounts. There is a time lag in the equilibration, and consequent creation of entropy. Now, however, the analog of the factor that produced the square of the electron-photon ratio in (1) need not give an extremely small number, since the numbers of ν 's and ϕ 's are comparable.

In principle, a formalism for the evolution of inhomogeneous

¹ In ref. [16] from 1987 it had already been noted that an interacting sea of neutrinos and majorons could form structure in the universe; of course the details of the interesting application of these ideas are now different, in view of all of the data that has come in since then.

geneities that followed the detailed evolution of the position and momentum distributions of each kind of particle would have no need at all to mention viscosity explicitly. Indeed, shear viscosity is subsumed within the standard evolution codes for the baryon, electron, photon systems, entering at the quadrupole level where non-isotropic stresses are taken into account. Bulk viscosity is not included, nor need it be, in the system of matter plus photons; but interacting neutrinos can be another story.

There are two important ways in which our $\nu - \phi$ system requires computational mechanics somewhat different from those necessary to derive the Weinberg formula:

1) In contrast to the case with matter and photons, where there is a conserved number of protons and electrons, the neutrino system will be taken to be symmetrical between particles and antiparticles, and the annihilation process into ϕ 's is an integral part of the evolution. This makes the determination of the analog of the factor in (1), $(\frac{1}{3} - [\frac{\partial p}{\partial \rho}]_n)$, different than in the photon-matter case.

2) Looking at the parameters for the problem at hand it appears that the frequency dependence of the bulk viscosity will play a role in the application. The consequences of this dependence have been important in the role of bulk viscosity in damping the gravitational radiation instabilities in rapidly rotating neutron stars [19]–[22]. This dependence, being essentially a non-local effect, does not rigorously fit into the framework that is best suited to the analysis of a relativistic fluid, *viz.* addition of an additional term in the energy-momentum tensor in the comoving system,

$$\delta T^{i,j} = -\delta^{i,j} \zeta(\omega) \nabla \cdot \mathbf{U}, \quad (2)$$

where \mathbf{U} is the space component of the four-velocity. We believe that it is legitimate, however, to crank the formalism forward using (2), and then, at the point when Fourier transforms are introduced in position space, to evaluate $\zeta(\omega)$ at $\omega = v_s k$ where v_s is the speed of sound in the medium.

In one respect, however, our $\nu - \phi$ system is simpler than the matter-photon system; the energy equilibration in a single collision is relatively efficient. This is because we have the direct transformation, $\nu + \nu \rightarrow \phi + \phi$.²

² In models that couple light scalars to neutrinos, the neutrinos might be taken either as Dirac neutrinos, with possible couplings that preserve lepton number, leading to $\nu + \bar{\nu} \rightarrow \phi + \phi$, or as Majorana neutrinos that violate lepton number, in which case we have $\nu + \nu \rightarrow \phi + \phi$. The amplitudes for the two cases are different (in angular dependence, for example) but we can take an estimate of total rate, as a function of coupling, G , to be the same in both cases. The distinction will not enter further in this paper, except here and there in terminology.

The formalism that we shall present is completely applicable to the case of photons and ordinary matter and this may possibly be of interest in other astronomical contexts. We shall apply our method to the ionized-hydrogen system, with a result related to (1)

CALCULATION

We consider an adiabatic compression specified by a value of $-\delta V/V$, where we do not allow energy transfer between the two species, leading to respective temperature changes for the species of $\delta T_{\phi,\nu}$. Defining the indices $\gamma_{\phi,\nu}$ for the respective species as,

$$\frac{\delta T_{\phi,\nu}}{T_{\phi,\nu}} = -\frac{\delta V}{V} (\gamma_{\phi,\nu} - 1), \quad (3)$$

we take

$$\frac{\delta V}{V} = b e^{i\omega t}, \quad (4)$$

where it is to be understood that we consistently take real parts to get linear perturbations in the physical quantities. The time rate of change of the temperatures is thus,

$$\frac{d}{dt} T_{\phi,\nu} = i b (\gamma_{\phi,\nu} - 1) \omega T_{\phi,\nu}. \quad (5)$$

The rate of heat transfer per unit volume between the two baths is of the order of,

$$\dot{Q} = \Gamma \rho_\phi \frac{(T_\phi - T_\nu)}{T_0}, \quad (6)$$

where T_0 is the mean temperature of both baths, ρ_ϕ the energy density of the ϕ particle bath, and Γ an effective collision rate for ϕ 's in the medium. By “effective” we mean the rate for collisions of a ϕ with the ν cloud, supplemented by rate for $\phi + \phi \rightarrow 2\nu$, and multiplied by an energy transfer efficiency factor which is of the order of, but less than, unity, (and model dependent as well). Adding the effect of heat transfer to the right hand side of (5) gives,

$$\begin{aligned} \frac{d}{dt} T_\phi(t) &= i(\gamma_\phi - 1) b \omega T_0 e^{i\omega t} - \Gamma \rho_\phi \left[\frac{T_\phi(t) - T_\nu(t)}{T_0} \right] [c_V^{(\phi)}]^{-1}, \\ \frac{d}{dt} T_\nu(t) &= i(\gamma_\nu - 1) b \omega T_0 e^{i\omega t} + \Gamma \rho_\phi \left[\frac{T_\phi(t) - T_\nu(t)}{T_0} \right] [c_V^{(\nu)}]^{-1}. \end{aligned} \quad (7)$$

Subtracting, defining $\delta T(t) = T_\phi(t) - T_\nu(t)$, and solving, we obtain,

$$\delta T(t) = \text{Re} \left[\frac{i\omega(\gamma_\phi - \gamma_\nu) T_0 b}{i\omega + \Gamma \rho_\phi ([c_V^{(\phi)}]^{-1} + [c_V^{(\nu)}]^{-1}) T_0^{-1}} e^{i\omega t} \right]. \quad (8)$$

In the calculation of work, $\int P dV$, below, the volume differential will be given by $[d/dt(\delta V)]dt = b V \omega \sin(\omega t)$.

We will therefore retain only the $\sin(\omega t)$ terms in the δT 's, since the $\cos(\omega t)$ terms do not contribute to the dissipation. We denote these parts and the associated pressure changes by δT^s , δP^s .

From (7) we then find that the $\sin(\omega t)$ term in the oscillation of the temperature difference is divided between the two baths as,

$$\begin{aligned}\delta T_\phi^s(t) &= \delta T^s(t) \frac{[c_V^{(\phi)}]^{-1}}{[c_V^{(\phi)}]^{-1} + [c_V^{(\nu)}]^{-1}}, \\ \delta T_\nu^s(t) &= -\delta T^s(t) \frac{[c_V^{(\nu)}]^{-1}}{[c_V^{(\phi)}]^{-1} + [c_V^{(\nu)}]^{-1}}.\end{aligned}\quad (9)$$

Next we calculate the induced pressure changes of the two clouds,

$$\begin{aligned}\delta P_\phi^s(t) &= \delta T_\phi^s(t) \left[\frac{dP_\phi}{dT} \right] \equiv \alpha_\phi \delta T_\phi^s(t), \\ \delta P_\nu^s(t) &= \delta T_\nu^s(t) \left[\frac{dP_\nu}{dT} \right] \equiv \alpha_\nu \delta T_\nu^s(t).\end{aligned}\quad (10)$$

Taking the ϕ mass to be zero, we list the three relevant parameters for the ϕ cloud,

$$\alpha_\phi = \frac{2\pi^2 T_0^3}{45}, \quad c_V^{(\phi)} = 3\alpha_\phi, \quad \gamma_\phi = 4/3, \quad (11)$$

where we use units $\hbar = c = k_B = 1$. For the neutrino cloud in the domain in which the mass plays a role, numerical calculations of the corresponding parameters are described in the appendix.

We write the net pressure oscillation amplitude as, $\delta P_\phi^s + \delta P_\nu^s$, obtaining,

$$\delta P^s \equiv \delta P_\phi^s + \delta P_\nu^s = -\omega b \zeta \sin \omega t, \quad (12)$$

where

$$\zeta = \frac{(\gamma_\nu - \gamma_\phi) \Gamma \rho_\phi}{\omega^2 + \Gamma^2 \rho_\phi^2 ([c_V^{(\phi)}]^{-1} + [c_V^{(\nu)}]^{-1})^2 T_0^{-2}} \left[\frac{\alpha_\nu}{c_V^{(\nu)}} - \frac{1}{3} \right]. \quad (13)$$

The amount of mechanical energy converted to thermal energy in one period is given by

$$\Delta E = \int_0^{2\pi/\omega} dt 2\zeta \omega \sin(\omega t) dV(t) = 2\pi \omega b^2 \zeta, \quad (14)$$

giving the time averaged dissipation per unit volume,

$$\dot{E}/V = \zeta \omega^2 b^2 / 2, \quad (15)$$

identifying ζ as the bulk viscosity.

Using eq.(5) of ref.[18] and staying in the temperature region that is (nearly) non-relativistic for the ν 's we calculate the reaction rate for $\phi + \phi \rightarrow \nu + \nu$ in the mixture

as $\Gamma = G^4 T g(m_\nu/T)$, where G is the $\nu - \chi$ coupling constant, and the dimensionless function $g(m_\nu/T)$ is given in the appendix. Introducing $k \approx 3\omega$, we write ζ in the form,

$$\zeta = \frac{10^4 G^{-4} \rho_\phi T_0^{-1} H\left(\frac{m_\nu}{T_0}\right)}{1 + k^2/k_0^2}, \quad (16)$$

where

$$H = \frac{10^{-4} (\gamma_\nu - 1/3) \left(\frac{\alpha_\nu}{c_V^{(\nu)}} - 1/3 \right) T_0^2}{g(m_\nu/T_0) \left([c_V^{(\phi)}]^{-1} + [c_V^{(\nu)}]^{-1} \right)^2 \rho_\phi^2}. \quad (17)$$

The ‘‘cutoff’’ wave number k_0 is given as,

$$k_0 = 10^{-3} G^4 T_0 F\left(\frac{m_\nu}{T_0}\right), \quad (18)$$

where

$$\begin{aligned}F\left(\frac{m_\nu}{T_0}\right) &= 10^3 [g(m_\nu/T_0)]^{1/2} 3^{-1/2} \\ &\times \rho_\phi T_0^{-1} \left([c_V^{(\phi)}]^{-1} + [c_V^{(\nu)}]^{-1} \right).\end{aligned}\quad (19)$$

F and H are dimensionless functions of m_ν/T_0 , plotted in figs. 1 and 2 for the range of values in which we are most interested. They are both of order unity in the semirelativistic region in which there may be significant effects of bulk viscosity. The calculation of the components of the above functions, α_ν , γ_ν , and g is discussed in the appendix.

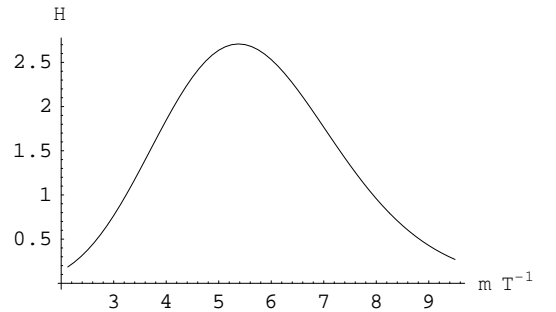


FIG. 1: The function $H(m_\nu/T)$ which appears in the bulk viscosity result, (16).

The damping rate for a plane sound wave in a medium with bulk viscosity, ζ , is given by eq.(2.56) of ref.[1],

$$\Gamma_{\text{damp}} = \frac{k^2 \zeta}{2(\rho + p)}. \quad (20)$$

We divide by the frequency for a sound wave of wave-number k , take $v_s \approx 1/\sqrt{3}$, and shift to appropriate units,

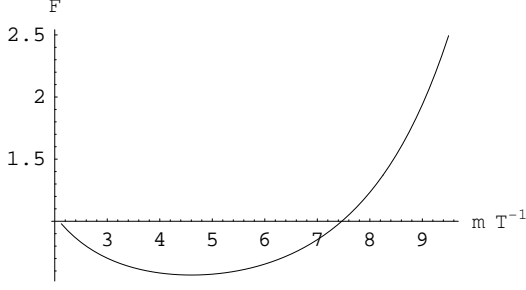


FIG. 2: The function $F(m_\nu/T)$ which appears in the result for the wave-number cutoff k_0 (18).

obtaining,

$$\Gamma_{\text{damp}}/(kv_s) \approx .28 \left(\frac{k}{[1 \text{ Mpc}]^{-1}} \right) \left(\frac{10^{-6}}{G} \right)^4 \left(\frac{.2 \text{ eV}}{T_0} \right) \times H \left(\frac{m_\nu}{T_0} \right) (1 + k^2/k_0^2)^{-1}. \quad (21)$$

where

$$\frac{k_0}{[1 \text{ Mpc}]^{-1}} = 30.3 \left(\frac{G}{10^{-6}} \right)^4 \left(\frac{T_0}{.2 \text{ eV}} \right) F \left(\frac{m_\nu}{T_0} \right). \quad (22)$$

From (21) and (22) we see that there can be strong damping in several oscillations when the system is at the recombination temperature, for a neutrino mass that is of the order of 1 eV and coupling G of the order 10^{-6} .

MATTER PLUS PHOTON SYSTEMS.

All of our development is applicable to the system of ionized hydrogen considered by Weinberg, and we compare our result for this case to (1) in the low-frequency limit, $\zeta(\omega = 0)$. This will provide both a check on our pedestrian formalism, and possible application to ionized hydrogen systems at much higher densities and temperatures than prevail in the recombination region in cosmology. In (13), replacing the subscript ϕ by γ and ν by “mat”, we use the adiabatic indices for the matter, $\gamma_{\text{mat}} = 5/3$, $\alpha_{\text{mat}} = 2n_e$ and the specific heat, $c_V^{(\text{mat})} = 3n_e$ where n_e is the density of the protons and the electrons. For the photons we use the numbers from (11) but with α and C_V doubled, since there are two polarization states. We obtain,

$$\zeta(0) = \frac{16aT^4 n_e^2}{\Gamma(4aT^3 + 3n_e)^2}, \quad (23)$$

with $a = \pi^2/15$, the black-body constant in our units. The Weinberg formula (1) yields the same result if we choose $\Gamma = 4\tau_\gamma^{-1}$. We can check the relation between these parameters, in a system in which photon emission

and absorption (not scattering) provides the mechanism for energy transfer, and in which the photon absorption rate is independent of energy, by beginning from the Boltzmann equation for the photon distribution function $f(\omega)$,

$$\frac{\partial}{\partial t} f = \tau_\gamma^{-1} [(1 - e^{-\omega/T_m})f - e^{-\omega/T_m}]. \quad (24)$$

We estimate the rate of energy transfer \dot{Q} and from it the coefficient Γ in (6) by inserting $f = (\exp[\omega/T_\gamma - 1])^{-1}$, expanding to first order in $(T_\gamma - T_m)$, multiplying by ω and integrating d^3k . We obtain,

$$\Gamma = \frac{1}{\tau_\gamma W(T_0)} \int_0^\infty d\omega \omega^4 \frac{1}{[\exp(\omega/T_0) - 1]} = 3.83 \tau_\gamma^{-1}, \quad (25)$$

where,

$$W(T_0) = \int d\omega \omega^3 \frac{1}{\exp(\omega/T_0) - 1}. \quad (26)$$

We can directly apply (23) to determine bulk viscosity in regions in which it is dominated by the “free-free” photoemission and photoabsorption process, rather than by Compton scattering. This process is completely efficient in energy transfer, so that our previous caveats with respect to an efficiency factor do not apply. The results will be most interesting in a system in which we have neither $n_e \gg n_\gamma$, nor $n_e \ll n_\gamma$. We take the photo-absorption rate, Γ_{abs} from the Kramers formula, modified by the Gaunt factor correction [23];

$$\Gamma_{\text{abs}}(\omega) = \frac{16\pi e^6 n_e^2}{3m_e \omega^3} \sqrt{\frac{2\pi}{m_e T}} e^{\omega/(2T)} K_0\left(\frac{\omega}{2T}\right), \quad (27)$$

where K_0 is the modified Bessel function.

Defining the parameter to be used in this case for Γ in (6) by Γ_{ff} , we have, in place of (25),

$$\begin{aligned} \Gamma_{ff} &= \frac{1}{W(T_0)} \int_0^\infty d\omega \Gamma_{\text{abs}}(\omega) \frac{\omega^4}{\exp(\omega/T_0) - 1} \\ &= 114. e^6 m_e^{-3/2} T_0^{-7/2} n_e^2. \end{aligned} \quad (28)$$

Taking T/m_e as the efficiency factor for energy transfer in the Compton process, we find that the bulk viscosity is determined by Γ_{ff} , rather than Compton scattering in all regions except those in which $n_e \ll T^3$. In the latter regions, however it is much smaller than the shear viscosity by virtue of the n_e dependence of (23).

DISCUSSION

The principal result of this paper is the expression for the bulk viscosity of the interacting gas of ν 's and ϕ 's. We

took one neutrino flavor, and an initial state with vanishing chemical potential, i. e. equal numbers of ν 's and $\bar{\nu}$'s (or equal numbers of right-handed and left-handed neutrinos in the Majorana variation).

For the tightly coupled case with, say, $G \approx 10^{-5}$, we would find very little dissipation in our models. For couplings of $G \approx 10^{-6}$, and a neutrino mass of order of an eV we find relatively strong damping of a plane sound wave in the medium. We note the following about this regime:

1) In the simplest coupling schemes, at least, there would be a negligible number of ϕ 's produced prior to neutrino-matter decoupling. But for $G \approx 10^{-6}$ there would be production at a rate that serves to create an equilibrium distribution during the period in which the neutrinos are still completely relativistic and prior to photon decoupling.

2) In this domain, the shear viscosity and bulk viscosity will make comparable contributions to the damping of a plane wave. However, when the shift is made to a proper cosmological calculation, in which the shear viscosity is implicit in the way the scattering rate enters the quadrupole terms, and the bulk velocity damps monopoles, it is less clear to us what the relative contributions of bulk viscosity to the actual signal will be.

As we move into the region $G < 10^{-6}$ the dissipation becomes greater, and it appears that our effects would be an essential element of the analysis of the perturbations. We note that the authors of ref [17] found that their free-streaming criterion ruled out couplings down to a value $G \approx 10^{-7}$.

Finally, as will have been obvious, we made no attempt to include the complications that would arise from three flavors of neutrinos with a mass splitting scheme to agree with the data, let alone the possible complications of ϕ couplings that carry anything other than the unit matrix in flavor space. When couplings that allow ν decays are included, the basic physics of our description remains unchanged, but the numerology concerning the interesting domain of G 's becomes very different.

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APPENDIX

We need the temperature response $\gamma_\nu - 1$ to an adiabatic change in volume, and the pressure response α_ν to this temperature response, all of this with the interactions between ν and ϕ that transfer energy between the baths turned off. A complication is that in the true equilibrium state, in the presence of the annihilation mechanism, the number of ν 's and $\bar{\nu}$'s (the latter, if we use Dirac ν 's) are not conserved. The average state, to which we apply our periodic disturbance, thus retains the value $\mu = 0$ for neutrino chemical potentials. But to conserve

neutrino (and anti-neutrino, if we use Dirac ν 's) number in our periodic changes we need to introduce an oscillating chemical potential perturbation $\delta\mu_\nu$ as well as an oscillating value for $\delta V/V$. For the Dirac neutrino case the $\delta\mu_s$ are the *same* for neutrino and antineutrino (in contrast to an equilibrium case, where they would be equal and opposite if there were zero net lepton number.) We now have an energy equation,

$$(P + \rho) \frac{\delta V}{V} = - \frac{\partial \rho_\nu}{\partial T} \delta T - \frac{\partial \rho_\nu}{\partial \mu} \delta \mu_\nu, \quad (29)$$

and a number conservation equation,

$$\frac{\delta V}{V} = - \frac{1}{n_\nu} \left[\frac{\partial n_\nu}{\partial T} \delta T + \frac{\partial n_\nu}{\partial \mu_\nu} \delta \mu \right]. \quad (30)$$

where μ_ν is to be evaluated at zero after differentiation. Defining,

$$A = \left[\frac{1}{\rho_\nu + p_\nu} \frac{\partial \rho_\nu}{\partial T} - \frac{1}{n} \frac{\partial n}{\partial T} \right] \left[\frac{1}{n} \frac{\partial n}{\partial \mu} - \frac{1}{p + \rho_\nu} \frac{\partial \rho_\nu}{\partial \mu_\nu} \right]^{-1}, \quad (31)$$

we have

$$\delta \mu_\nu = A \delta T, \quad (32)$$

$$\gamma_\nu - 1 = \left[\frac{\partial \rho_\nu}{\partial T} + A \frac{\partial \rho_\nu}{\partial \mu_\nu} \right] [p_\nu + \rho_\nu]^{-1},$$

$$\alpha_\nu = \frac{\partial p_\nu}{\partial T}. \quad (33)$$

The quantities above are calculated from the expressions,

$$n_\nu = \frac{1}{\pi^2} \int_0^\infty p^2 dp \frac{1}{1 + e^{(E - \mu_\nu)/T}}, \quad (34)$$

$$\rho_\nu = \frac{1}{\pi^2} \int_0^\infty p^2 dp \frac{E}{1 + e^{(E - \mu_\nu)/T}}, \quad (35)$$

$$p_\nu = \frac{1}{\pi^2} \int_0^\infty p^2 dp \frac{p^2/E}{1 + e^{(E - \mu_\nu)/T}}, \quad (36)$$

where $E = \sqrt{p^2 + m_\nu^2}$. The parameters $\gamma_\nu - 1$, α_ν are functions of the single variable, T/m_ν . In figs. 3 and 4. we show plots of the two parameters.

Note that γ_ν approaches 4/3 at high temperatures, as expected. We note a borderline inconsistency in our procedures, above, in that we will trust interactions that equilibrate the neutrino distribution thermally, such as $\nu - \nu$ scattering (from ϕ exchange) to do their job, while turning off (schematically) the change in neutrino number and transfer of energy between the two seas, although all of these effects are of the same order in G .

For the function $g(m_\nu/T)$ we take the expression given in eq(5) of ref. [14], for the ν annihilation rate, and

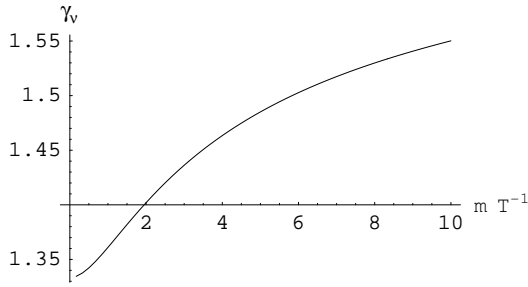


FIG. 3: The index $\gamma_\nu(m_\nu/T)$.

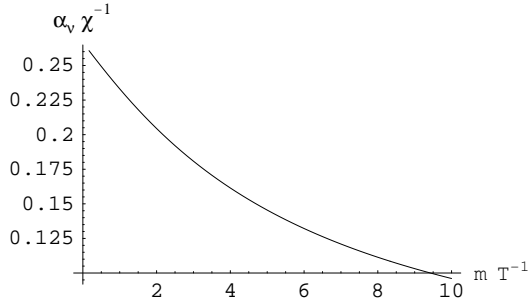


FIG. 4: The function $\alpha_\nu(m_\nu/T)$ expressed as a multiple of the specific heat $\chi = c_V$.

multiply by the appropriate density ratio to obtain the estimate for the rate for $\phi + \phi \rightarrow \nu + \nu$,

$$\Gamma = \frac{G^4}{64\pi} \frac{T}{m_\nu^3} \left(\frac{m_\nu T}{2\pi} \right)^{3/2} \frac{n_\phi}{n_\nu} e^{-m_\nu/T}. \quad (37)$$

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